To complete the preprint titled "An affirmative answer to a conjecture for Metoki class by Kentaro Mikami at Akita University", we asked help of Groebner Basis Package of Maple, which is one of Symbol Calculus Softwares, in order to get bases of given cochain complexes and the cohomogoly groups.
In the preprint above, we have shown a basis of $\mathrm{H}^7_{\mathrm{GF}}(\mathfrak{ham}_2, \mathfrak{sp}(2,\mathbb{R}))_8)$ concretely by Groebner Basis Package of Maple.
In this note, we do the same jobi by using Risa/Asir, which is another Symbol Calculus Software, and show that the results we got by Maple are the same up to non-zero scalar multiples.
We remark that in this note we added some line breaks so that we get better look and we use `nd_gr()` instead of `gr()`.
We stock two matrix representations of $d_0$ in the two files:

**Mat_w8_6and7_type0.rr**

```
G1 = 135/4*y1-20*y3-60*y4+15*y5+5/4*y6-135/4*y7-15*y8-15*y9$
G2 = -18*y1-12*y2+23/3*y6-45*y7+21*y8+10*y9$
G3 = -27/4*y1-20*y3+12*y4+3*y5+63/8*y10-18*y11$
G4 = 2*y3-10*y4+2/3*y5+y6+2*y7-1/3*y8+2*y9-7*y10+y11$
G5 = -11/2*y2-17*y3+9*y4+71/12*y5+7/8*y6+51/8*y7+233/12*y8+10*y9-49/4*y10+4*y11$
G6 = -1/3*y2-9*y3+5*y4+49/18*y5+5/12*y6+55/4*y7+47/9*y8+19/3*y9-28/3*y10-1/6*y11$
G7 = -5/2*y2-20*y3+65/12*y5-15/8*y6-75/8*y7+65/12*y8-5*y9+35/4*y10-25/2*y11$
G8 = 7*y6-18*y7-9*y8+6*y9$
G9 = -2*y3-6*y4-21/2*y10+6*y11+70*y12$
G10 = -1/2*y1+2*y2-2*y3+14/3*y4+1/3*y5-28*y13-14/3*y14$
G11 = -5/2*y1+2*y2+16/3*y4-1/3*y5-28*y13-28/3*y14$
G12 = 8*y2-3*y6+y8-6*y9-112*y13$
G13 = -y6+15*y7-6*y8-3/2*y10-3*y11+48*y13-14*y14$
G14 = -2/3*y6-y7-y8-2*y9+5/2*y10-y11+4*y13-14/3*y14$
G15 = 4/3*y5+11/3*y6-45*y7+46/3*y8+5/2*y10+11*y11-136*y13+14*y14$
G16 = 2*y3-y5-3/2*y6+123/8*y7-4*y8-21/16*y10-33/16*y11+42*y13-21/8*y14$
G17 = 2*y4-1/3*y5-1/2*y6+39/8*y7-4/3*y8+7/16*y10-13/16*y11+14*y13-35/24*y14$
G18 = y10-y11-20/3*y12+2*y14$
    GList = [G1,G2,G3,G4,G5,G6,G7,G8,G9,C10,G11,G12,G13,G14,G15,G16,G17,G18]$
    YList = [y1,y2,y3,y4,y5,y6,y7,y8,y9,y10,y11,y12,y13,y14] $
GG1 = 135/4*F1-20*F3-60*F4+15*F5+5/4*F6-135/4*F7-15*F8-15*F9$
GG2 = -18*F1-12*F2+23/3*F6-45*F7+21*F8+10*F9$
GG3 = -27/4*F1-20*F3+12*F4+3*F5+63/8*F10-18*F11$
GG4 = 2*F3-10*F4+2/3*F5+F6+2*F7-1/3*F8+2*F9-7*F10+F11$
GG5 = -11/2*F2-17*F3+9*F4+71/12*F5+7/8*F6+51/8*F7+233/12*F8+10*F9-49/4*F10+4*F11$
GG6 = -1/3*F2-9*F3+5*F4+49/18*F5+5/12*F6+55/4*F7+47/9*F8+19/3*F9-28/3*F10-1/6*F11$
GG7 = -5/2*F2-20*F3+65/12*F5-15/8*F6-75/8*F7+65/12*F8-5*F9+35/4*F10-25/2*F11$
GG8 = 7*F6-18*F7-9*F8+6*F9$              GG9 = -2*F3-6*F4-21/2*F10+6*F11+70*F12$
GG10 = -1/2*F1+2*F2-2*F3+14/3*F4+1/3*F5-28*F13-14/3*F14$
GG11 = -5/2*F1+2*F2+16/3*F4-1/3*F5-28*F13-28/3*F14$
GG12 = 8*F2-3*F6+F8-6*F9-112*F13$
GG13 = -F6+15*F7-6*F8-3/2*F10-3*F11+48*F13-14*F14$
GG14 = -2/3*F6-F7-F8-2*F9+5/2*F10-F11+4*F13-14/3*F14$
GG15 = 4/3*F5+11/3*F6-45*F7+46/3*F8+5/2*F10+11*F11-136*F13+14*F14$
GG16 = 2*F3-F5-3/2*F6+123/8*F7-4*F8-21/16*F10-33/16*F11+42*F13-21/8*F14$
GG17 = 2*F4-1/3*F5-1/2*F6+39/8*F7-4/3*F8+7/16*F10-13/16*F11+14*F13-35/24*F14$
GG18 = F10-F11-20/3*F12+2*F14$
     GGList = [GG1,GG2,GG3,GG4,GG5,GG6,GG7,GG8,GG9,C10,GG11,GG12,GG13,GG14,GG15,
               GG16,GG17,GG18]$
```

**Mat_w8_7and8_type0.rr**.

```
F1 = 11*w2-16*w3$                 F2 = -35*w1-9*w2+8*w3$
F3 = -39/8*w2+9*w3+63/2*w4$       F4 = -31/8*w2+5*w3+21/2*w4$
F5 = -61/2*w2+52*w3+84*w4$        F6 = -30*w1-75*w2+20*w3$
F7 = -10*w2+8/3*w3$               F8 = -15*w1-10*w2-2*w3$
F9 = 25/2*w1+85/2*w2-55/3*w3$     F10 = -2/3*w2$
F11 = -20/3*w2+8*w3-14*w4$        F12 = 3*w4$
F13 = -5/2*w1-w2+w3$             F14 = -3*w2+4*w3+3*w4$

FList = [F1,F2,F3,F4,F5,F6,F7,F8,F9,F10,F11,F12,F13,F14]$
WList = [w1,w2,w3,w4]$
YList = [y1,y2,y3,y4,y5,y6,y7,y8,y9,y10,y11,y12,y13,y14]$
CList = [c1,c2,c3,c4,c5,c6,c7,c8,c9,c10,c11,c12,c13,c14]$
```

**Basis of** $d_0\left(\mathbf{C}^6_{\mathrm{GF}}(\mathfrak{ham}_2, \mathfrak{sp}(2,\mathbb{R}))_8\right) \subset \mathbf{C}^7_{\mathrm{GF}}(\mathfrak{ham}_2, \mathfrak{sp}(2,\mathbb{R}))_8$**:**
Our source file for Risa/Asir is this:

```
load("./Mat_w8_6and7_type0.rr")$ /* GB1 = gr( [ ], [ ], 0) $ */
ord( YList ) $
GB1 = reverse(
nd_gr(GList , YList ,0,0)) $
print(["GBe",GB1])$
end$
```

The output of Groebner Basis is the next:

```
[GBe, [3*y10-3*y11-20*y12+6*y14, 100*y8+36*y9-15*y11-420*y12-420*y13+350*y14,
      -300*y7-84*y9+135*y11+980*y12-420*y13-350*y14,
       100*y6+204*y9-135*y11-1380*y12-180*y13+750*y14,
       40*y5-12*y9+15*y11-460*y12-60*y13-590*y14,
       4800*y4+84*y9+2565*y11+6020*y12+420*y13-10850*y14,
       1600*y3-84*y9+1035*y11-6020*y12-420*y13-5950*y14,
       400*y2-12*y9-195*y11-1860*y12-5660*y13+950*y14,
      -450*y1-24*y9-315*y11-220*y12-120*y13-1250*y14]]
```

**Kernel space of** $d_0 : \mathbf{C}^7_{\mathrm{GF}}(\mathfrak{ham}_2, \mathfrak{sp}(2,\mathbb{R}))_8 \to \mathbf{C}^8_{\mathrm{GF}}(\mathfrak{ham}_2, \mathfrak{sp}(2,\mathbb{R}))_8$**:**
Our source file for Risa/Asir is this:

```
load("Mat_w8_7and8_type0.rr")$  NagasaW = length(WList)$  NagasaF = length(FList)$
for ( Uke = [], J=1; J <= NagasaW; J++ ) { MyA = WList[J-1];  Atai = 0;
    for (K=1 ; K <= NagasaF; K++ ){ MyB = FList[K-1];
    Atai += diff( MyB, MyA)* CList[K-1];}
    Uke = cons(Atai, Uke ); }
print("mark A")$    Uke = reverse( Uke );
print("mark B")$    GBadj = nd_gr( Uke, CList, 0, 0 );
for (H=0, I=1; I <= NagasaF; I++){ H += CList[I-1]* YList[I-1]; }
Hnf = p_nf(H, GBadj, CList, 0)$
for( MyUkez = [], T=CList; T != []; T = cdr(T)){
    MyA = car(T); MyV = diff( Hnf, MyA);
    MyUkez = cons( MyV, MyUkez);;}
```

```
print("mark C")$    MyUkez = reverse(MyUkez);
ord(YList)$
print("mark D")$ GBk = reverse( nd_gr( MyUkez, YList, 0, 0 ));
end$
```

The outputs are the follows:

```
mark A
[-35*c2-30*c6-15*c8+25/2*c9-5/2*c13,
11*c1-9*c2-39/8*c3-31/8*c4-61/2*c5-75*c6-10*c7
    -10*c8+85/2*c9-2/3*c10-20/3*c11-c13-3*c14,
-16*c1+8*c2+9*c3+5*c4+52*c5+20*c6+8/3*c7-2*c8-55/3*c9+8*c11+c13+4*c14,
63/2*c3+21/2*c4+84*c5-14*c11+3*c12+3*c14]
mark B
[-168*c1+336*c5-1260*c6-168*c7-294*c8+525*c9-16*c10+112*c11-12*c12+3*c13+24*c14,
-14*c2-12*c6-6*c8+5*c9-c13,
-126*c3-420*c5+2796*c6+392*c7+474*c8-1375*c9+32*c10+84*c11-6*c12+3*c13+6*c14,
-42*c4+84*c5-2796*c6-392*c7-474*c8+1375*c9-32*c10-28*c11-6*c12-3*c13-18*c14]
mark C
[0, 0, 0, 0,
1008*y1-1680*y3+1008*y4+504*y5,
-3780*y1-432*y2+11184*y3-33552*y4+504*y6,
-504*y1+1568*y3-4704*y4+504*y7,
-882*y1-216*y2+1896*y3-5688*y4+504*y8,
1575*y1+180*y2-5500*y3+16500*y4+504*y9,
-48*y1+128*y3-384*y4+504*y10,
336*y1+336*y3-336*y4+504*y11,
-36*y1-24*y3-72*y4+504*y12,
9*y1-36*y2+12*y3-36*y4+504*y13,
72*y1+24*y3-216*y4+504*y14]
mark D
[-3*y10+3*y11+20*y12-6*y14,
-12*y9-495*y11-3260*y12-60*y13+950*y14,
y8-15*y11-102*y12-6*y13+32*y14,
3*y7-36*y11-238*y12+70*y14,
-2*y6+171*y11+1136*y12+24*y13-338*y14,
4*y5+51*y11+280*y12-154*y14,
16*y4-3*y11-56*y12-14*y14,
16*y3+45*y11+168*y12-126*y14,
4*y2+3*y11+14*y12-56*y13,
-2*y1+3*y11+28*y12-14*y14]
```

**Basis of $\mathbf{H}^7_{\mathrm{GF}}(\mathfrak{ham}_2, \mathfrak{sp}(2,\mathbb{R}))_8$**

The next is a source file for Risa/Asir. GBe and GBk are data gotten above.

```
GBe = [ 3*y10-3*y11-20*y12+6*y14,
        100*y8+36*y9-15*y11-420*y12-420*y13+350*y14,
        -300*y7-84*y9+135*y11+980*y12-420*y13-350*y14,
        100*y6+204*y9-135*y11-1380*y12-180*y13+750*y14,
        40*y5-12*y9+15*y11-460*y12-60*y13-590*y14,
```

```
        4800*y4+84*y9+2565*y11+6020*y12+420*y13-10850*y14,
        1600*y3-84*y9+1035*y11-6020*y12-420*y13-5950*y14,
        400*y2-12*y9-195*y11-1860*y12-5660*y13+950*y14,
        -450*y1-24*y9-315*y11-220*y12-120*y13-1250*y14]$
GBk = [ -3*y10+3*y11+20*y12-6*y14,
        -12*y9-495*y11-3260*y12-60*y13+950*y14,
        y8-15*y11-102*y12-6*y13+32*y14,
        3*y7-36*y11-238*y12+70*y14,
        -2*y6+171*y11+1136*y12+24*y13-338*y14,
        4*y5+51*y11+280*y12-154*y14,
        16*y4-3*y11-56*y12-14*y14,
        16*y3+45*y11+168*y12-126*y14,
        4*y2+3*y11+14*y12-56*y13,
        -2*y1+3*y11+28*y12-14*y14]$
YList = [y1,y2,y3,y4,y5,y6,y7,y8,y9,y10,y11,y12,y13,y14]$
for(Uke=[], T= GBk; T != []; T = cdr(T)) {
    MyA = car(T); /* print(MyA); */
    Atai = p_nf( MyA, GBe, YList , 0) ; Uke = cons(Atai, Uke); }
Uke = reverse(Uke)$
ord(YList)$
GBb = reverse( nd_gr( Uke, YList , 0, 0) );
end$
```

A basis of $\mathrm{H}^7_{\mathrm{GF}}(\mathfrak{ham}_2, \mathfrak{sp}(2,\mathbb{R}))_8$ is given by the output

```
[-12*y9-495*y11-3260*y12-60*y13+950*y14]
```

We may omit the job of **Check** $d_0 \circ d_0$ : $\mathbf{C}^6_{\mathrm{GF}}(\mathfrak{ham}_2, \mathfrak{sp}(2,\mathbb{R}))_8 \to \mathbf{C}^8_{\mathrm{GF}}(\mathfrak{ham}_2, \mathfrak{sp}(2,\mathbb{R}))_8$ **is identically zero**.