To complete the preprint titled "An affirmative answer to a conjecture for Metoki class by Kentaro Mikami at Akita University", we asked help of Groebner Basis Package of Maple, which is one of Symbol Calculus Softwares, in order to get bases of given cochain complexes and the cohomogoly groups.
In this note, we make use of Risa/Asir, which is another Symbol Calculus Software, and show the results we got by Maple are the same up to non-zero scalar multiples.
We remark that we added some line breaks so that we get better look.

Basis of $d_1\left(\mathbf{C}^4_{\mathrm{GF}}(\mathfrak{ham}^0_2,\mathfrak{sp}(2,\mathbb{R}))_{10}\right)\subset \mathbf{C}^5_{\mathrm{GF}}(\mathfrak{ham}^0_2,\mathfrak{sp}(2,\mathbb{R}))_{10}$**:**
Our source file for Risa/Asir is this:

```
/* #####  On C^{4} -> C^{5} #### */
G1 = -135/4*y1-60*y3+15/2*y4-45*y5-15*y6+5/4*y7-45/4*y8+75/2*y9$
G2 = 108/11*y1+18/11*y2+60/11*y6+46/11*y7-90/11*y8+156/11*y9$
G3 = 27/4*y1+12*y3-9/2*y4-9*y5+27/4*y10+18*y11$
G4 = -10*y3+2/3*y4-2*y5+2*y6+y7+6*y9+4*y10-y11$
G5 = 5/2*y2+29*y3+47/3*y4-23*y5+43*y6+13/2*y7+9/2*y8+25*y9+16*y10-71/2*y11$
G6 = 5*y2+45*y3+155/6*y4-40*y5+65*y6+10*y7+50*y9+20*y10-115/2 *y11$
G7 = 3/2*y2+18*y3+23/2*y4-3*y5+30*y6+11/2*y7+9/2*y8+9*y9+6* y10-33*y11$
G8 = 6*y6+7*y7-6*y9$        G9 = -6*y3-3*y4+3*y10-6*y11+70*y12$
GBe = gr([G1,G2,G3,G4,G5,G6,G7,G8,G9],[y1,y2,y3,y4,y5,y6,y7,y8,y9,y10,y11,y12],1) ;
```

The output of Groebner Basis is the next:

```
[-140*y12+30*y11-15*y10-18*y9-9*y8+21*y7,
 140*y12-30*y11+15*y10+9*y8+18*y6,
 -3290*y12+1461*y11+666*y10+900*y9-75*y8-1512*y5,
 -770*y12+57*y11-18*y10+36*y9-3*y8+36*y4,
 70*y12-15*y11+18*y10+36*y9-3*y8-72*y3,
 3080*y12-660*y11-258*y10+396*y9-327*y8+63*y2,
 -1820*y12+390*y11+99*y10+144*y9-12*y8+189*y1]
```

Kernel space of $d_1 : \mathbf{C}^5_{\mathrm{GF}}(\mathfrak{ham}^0_2,\mathfrak{sp}(2,\mathbb{R}))_{10}\to \mathbf{C}^6_{\mathrm{GF}}(\mathfrak{ham}^0_2,\mathfrak{sp}(2,\mathbb{R}))_{10}$**:**
Our source file for Risa/Asir is this:

```
/* #####  On C^{5} -> C^{6} #### */
F1 = -5*w2-16*w3$                  F2 = 140*w1-4*w2+32*w3$
F3 = 1/4*w2-2*w3$                  F4 = -11/2*w2-12*w3+42*w4$
F5 = 31/12*w2+22/3*w3+7*w4$        F6 = -15*w1+31/6*w2+58/3*w3$
F7 = 15*w1-3*w2-18*w3$             F8 = 30*w1-2*w2-12*w3$
F9 = 5/2*w1+5/3*w2-5/3*w3$         F10 = -w2$
F11 = 2*w2+8*w3+14*w4$             F12 = 3*w4$
FList = [F1,F2,F3,F4,F5,F6,F7,F8,F9,F10,F11,F12]$   WList = [w1,w2,w3,w4]$
/* ############################################################ */
NagawaW = length(WList)$  NagasaF = length(FList)$
CC = [c1,c2,c3,c4,c5,c6,c7,c8,c9,c10,c11,c12]$
YY = [y1,y2,y3,y4,y5,y6,y7,y8,y9,y10,y11,y12]$
for ( Uke = [], J=1; J <= NagawaW; J++ ) { MyA = WList[J-1];  Atai = 0;
        for (K=1 ; K <= NagasaF; K++ ){ MyB = FList[K-1];
            Atai += diff( MyB, MyA)* CC[K-1];}; Uke = cons(Atai, Uke );
```

```
}
print("mark A")$    Uke = reverse( Uke );
print("mark B")$    GBadj = gr( Uke, CC, 0); /* Groebner Basis */
for (H=0, I=1; I <= NagasaF; I++){ H += CC[I-1]* YList[I-1]; }
Hnf = p_nf(H, GBadj, CC, 0)$  /* Normal Form */
for( MyUkez = [], T=CC; T != []; T = cdr(T)){
    MyA = car(T);   MyV = diff( Hnf, MyA); MyUkez = cons( MyV, MyUkez);}
print("mark C")$    MyUkez = reverse(MyUkez);
print("mark D")$    GBk = gr( MyUkez, YY, 0); /* Groebner Basis */
end$
```

The outputs are the follows:

```
mark A
[140*c2-15*c6+15*c7+30*c8+5/2*c9,
-5*c1-4*c2+1/4*c3-11/2*c4+31/12*c5+31/6*c6-3*c7-2*c8+5/3*c9-c10+2*c11,
-16*c1+32*c2-2*c3-12*c4+22/3*c5+58/3*c6-18*c7-12*c8-5/3*c9+8*c11,
42*c4+7*c5+14*c11+3*c12]
mark B
[42*c4+7*c5+14*c11+3*c12,
-42*c3-28*c5+114*c6-198*c7-228*c8-117*c9+48*c10-4*c11-6*c12,
56*c2-6*c6+6*c7+12*c8+c9,
168*c1-112*c5-182*c6+126*c7+84*c8-35*c9+24*c10-128*c11-12*c12]
mark C
[0, 0, 0, 0,
-168*y5+28*y4+112*y3-112*y1,    -168*y6-456*y3-18*y2-182*y1,
-168*y7+792*y3+18*y2+126*y1,    -168*y8+912*y3+36*y2+84*y1,
-168*y9+468*y3+3*y2-35*y1,      -168*y10-192*y3+24*y1,
-168*y11+56*y4+16*y3-128*y1,    -168*y12+12*y4+24*y3-12*y1]
mark D
[14*y12-3*y11-72*y10-36*y9+3*y8,        14*y12-3*y11+33*y10+18*y9-3*y7,
98*y12-21*y11+231*y10+108*y9+18*y6,     70*y12-33*y11+27*y10+36*y5,
42*y12-3*y11+5*y10-2*y4,                14*y12-3*y11-9*y10-12*y3,
-658*y12+141*y11+1158*y10+504*y9-9*y2,  28*y12-6*y11+3*y10-3*y1]
```

Basis of $\mathbf{H}^5_{\mathrm{GF}}(\mathfrak{ham}^0_2, \mathfrak{sp}(2,\mathbb{R}))_{10}$

The next is a source file for Risa/Asir. GBe and GBk are data gotten above.

```
GBe = [-140*y12+30*y11-15*y10-18*y9-9*y8+21*y7,
        140*y12-30*y11+15*y10+9*y8+18*y6,
       -3290*y12+1461*y11+666*y10+900*y9-75*y8-1512*y5,
       -770*y12+57*y11-18*y10+36*y9-3*y8+36*y4,
        70*y12-15*y11+18*y10+36*y9-3*y8-72*y3,
        3080*y12-660*y11-258*y10+396*y9-327*y8+63*y2,
       -1820*y12+390*y11+99*y10+144*y9-12*y8+189*y1]$
GBk = [ 14*y12-3*y11-72*y10-36*y9+3*y8,         14*y12-3*y11+33*y10+18*y9-3*y7,
        98*y12-21*y11+231*y10+108*y9+18*y6,     70*y12-33*y11+27*y10+36*y5,
        42*y12-3*y11+5*y10-2*y4,                14*y12-3*y11-9*y10-12*y3,
       -658*y12+141*y11+1158*y10+504*y9-9*y2,   28*y12-6*y11+3*y10-3*y1]$
YY = [y1,y2,y3,y4,y5,y6,y7,y8,y9,y10,y11,y12]$
```

```
for(Uke=[], T= GBk; T != []; T = cdr(T)) {
    MyA = car(T); Atai = p_nf( MyA, GBe, YY , 0) ;  /* NormalForm */
    Uke = cons(Atai, Uke); }
Uke = reverse(Uke)$
GBh = gr( Uke, YY , 0);  /* Groebner Basis */
```

A basis of $\mathrm{H}^5_{\mathrm{GF}}(\mathfrak{ham}^0_2, \mathfrak{sp}(2,\mathbb{R}))_{10}$ is given by the output

```
[-14*y12+3*y11+72*y10+36*y9-3*y8]
```

Check $d_1 \circ d_1 : \mathbf{C}^4_{\mathrm{GF}}(\mathfrak{ham}^0_2, \mathfrak{sp}(2,\mathbb{R}))_{10} \to \mathbf{C}^6_{\mathrm{GF}}(\mathfrak{ham}^0_2, \mathfrak{sp}(2,\mathbb{R}))_{10}$ is zero identically:
The next is a source file for Risa/Asir.

```
/* Feb 07, 2014 n=1, type 1, weight 10, C^{4} --> C^{5} --> C^{6} */
G1 = -135/4*y1-60*y3+15/2*y4-45*y5-15*y6+5/4*y7-45/4*y8+75/2*y9$
G2 = 108/11*y1+18/11*y2+60/11*y6+46/11*y7-90/11*y8+156/11*y9$
G3 = 27/4*y1+12*y3-9/2*y4-9*y5+27/4*y10+18*y11$
G4 = -10*y3+2/3*y4-2*y5+2*y6+y7+6*y9+4*y10-y11$
G5 = 5/2*y2+29*y3+47/3*y4-23*y5+43*y6+13/2*y7+9/2*y8+25*y9+16 *y10-71/2*y11$
G6 = 5*y2+45*y3+155/6*y4-40*y5+65*y6+10*y7+50*y9+20*y10-115/2 *y11$
G7 = 3/2*y2+18*y3+23/2*y4-3*y5+30*y6+11/2*y7+9/2*y8+9*y9+6* y10-33*y11$
G8 = 6*y6+7*y7-6*y9$
G9 = -6*y3-3*y4+3*y10-6*y11+70*y12$
/* The next data are gotten by replacing  y to F and G to GG  in the above.  */
GG1 = -135/4*F1-60*F3+15/2*F4-45*F5-15*F6+5/4*F7-45/4*F8+75/2*F9$
GG2 = 108/11*F1+18/11*F2+60/11*F6+46/11*F7-90/11*F8+156/11*F9$
GG3 = 27/4*F1+12*F3-9/2*F4-9*F5+27/4*F10+18*F11$
GG4 = -10*F3+2/3*F4-2*F5+2*F6+F7+6*F9+4*F10-F11$
GG5 = 5/2*F2+29*F3+47/3*F4-23*F5+43*F6+13/2*F7+9/2*F8+25*F9+16 *F10-71/2*F11$
GG6 = 5*F2+45*F3+155/6*F4-40*F5+65*F6+10*F7+50*F9+20*F10-115/2 *F11$
GG7 = 3/2*F2+18*F3+23/2*F4-3*F5+30*F6+11/2*F7+9/2*F8+9*F9+6* F10-33*F11$
GG8 = 6*F6+7*F7-6*F9$
GG9 = -6*F3-3*F4+3*F10-6*F11+70*F12$
/* ### On C^{5} -> C^{6}: */
F1 = -5*w2-16*w3$                F2 = 140*w1-4*w2+32*w3$
F3 = 1/4*w2-2*w3$                F4 = -11/2*w2-12*w3+42*w4$
F5 = 31/12*w2+22/3*w3+7*w4$      F6 = -15*w1+31/6*w2+58/3*w3$
F7 = 15*w1-3*w2-18*w3$           F8 = 30*w1-2*w2-12*w3$
F9 = 5/2*w1+5/3*w2-5/3*w3$       F10 = -w2$
F11 = 2*w2+8*w3+14*w4$           F12 = 3*w4$
/* ########################################## */
L = [GG1,GG2,GG3,GG4,GG5,GG6,GG7,GG8,GG9]$      print(L)$   end$
```

The output is as expectd.

```
[0,0,0,0,0,0,0,0,0]
```